The Speed Optimal Control for the Group of Autonomous Moving Objects

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Abstract[1]

The problem of control for the group of autonomous moving objects aimed to provide a certain three-dimensional configuration (rank) of the objects is considered in the paper. To solve the problem, it is proposed to use relay laws belonging to speed optimal control laws. To simplify the law synthesis, multiply connected object decomposition to several simply connected objects with unrelated vectors of state variables is provided.

1. Introduction

The problem of control laws synthesis for the group of autonomous moving objects (group control problem) is really urgent now. It is caused by intelligent control systems advantage that realizes conceptually new and more efficient control algorithms [1].

The research is devoted to the group control for autonomous moving objects aimed to provide a certain three-dimensional rank of the objects. An example is providing a required three-dimensional rank of aircrafts (manned as well as unmanned) [2]. Automatic control systems (ACS) synthesis is rather complex because the systems have supervisory structure as a rule. At this rate, relative coordinate control of the objects is realized by their relative speed variation. It is provided by variations (nominal condition bend) of speed ACS master controls. In this case, using traditional linear laws for the group control leads to significant dynamic errors of the objects speed control. The errors values are in direct proportion to the objects' relative coordinates.

In the paper, it is introduced to use relay control laws for the group control systems for autonomous moving objects. The relay controls belong to the speed optimal control laws based on L.S. Pontryagin's principle of the maximum. Using these laws allows changing (in compare to nominal values) the parameters of the speed control system in desirable (in the sense of dynamic accuracy) interval with no dependence on control errors of the objects' relative coordinates.

A decomposition procedure is provided to overcome the difficulties occurring in determining such the control lows for multiply connected systems of the group control. It means the synthesis problem sequential decision for the considered controls laws of simply connected systems.

2. Problem Definition

Let's consider the control group system for the planar motion of N autonomous moving objects. Herein the controlled object is a multiply connected system of the related speed control for the group of moving objects having a certain configuration (rank). The control system is provided with relative coordinates sensors. The controlled object is considered fully Kalman controllable and described by vector-matrix equation

$$\bar{z}(s) = H(s)\bar{u}(s) , \tag{1}$$

where $\bar{z}(s)$ and $\bar{u}(s)$ are Laplace representations of n-dimensional output vectors (about Cartesian coordinates

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of the moving objects) and control actions, $n = N$; $H(s)$ is a transfer function defined as

$$H(s) = \frac{1}{d(s)} M(s) \,, \qquad (2)$$

where $d(s) = \sum_{i=1}^{m} d_i s^i$; $M(s) = [\mu_{ij}(s)]_{n \times n}$ is a square matrix, the elements of which are polynomials $\mu_{ij}(s)$ (the order of μ_{ij} is less than m for all of i and j). The descriptions "input-output" (1), (2) may be associated (using the certain approach [3]) with the controlled object description with Cauchy's equations about state variables

$$\dot{\overline{x}} = A\overline{x} + B\overline{u} \,, \qquad (3)$$

where $\overline{x} = [\overline{x}_1, \ldots, \overline{x}_n]^{\mathrm{T}}$ is a block vector of the object's state variables, moreover the first component $x_{1i} \, (i = \overline{1,n})$ of the state variables vector $\overline{x}_i$ (the vector dimension is m for all of i) is i-th output coordinate of the multiply connected object; $\overline{u} = [u_1, \ldots, u_n]^{\mathrm{T}}$ is n-dimensional vector of control actions with the following constraints on it's components

$$| u_i(t) | \leq a_i \,, \; i = \overline{1,n} \,, \qquad (4)$$

where a_i value is determined from a permissible action value on a speed control channel for the associated moving object; A matrix is a block-diagonal matrix

$$A = \begin{bmatrix} A_0 & \cdots & 0 \\ \cdots & \ddots & \cdots \\ 0 & \cdots & A_0 \end{bmatrix}_{(nm)\times(mn)} \,, \qquad (5)$$

where

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ d_0 & d_1 & d_2 & \cdots & d_m \end{bmatrix}_{m \times m} \,, \qquad (6)$$

and

$$\det A = \det(Is - A_0)^n = [d(s)]^n \qquad (7)$$

is the field equation of the controlled object (I is the a unit matrix); $B = [B_1, \ldots, B_n]^{\mathrm{T}}$ is a block column matrix $(nm) \times m$ dimensioned where elements B_i are respectively grouped [3] polynomial coefficients $\mu_{ij}(s)$ and $d(s)$, herein all the matrixes B_i are (nm) dimensioned and rank $B = n$.

Let us assign initial $\overline{x}(t_{\mathrm{In}}) = \overline{x}^{\mathrm{In}}$ and finite $\overline{x}(t_{\mathrm{F}}) = \overline{x}^{\mathrm{F}}$ conditions of the state variables vector where t_{In} and t_{F} are initial and finite time points accordingly.

It is required to determine control laws $u_i(t)$, $i = \overline{1,n}$ that belong to the speed optimal control laws and take the vector condition variables from a given state $\overline{x}^{\mathrm{In}}$ to a given finite condition $\overline{x}^{\mathrm{F}}$ under fulfilling constraints (4).

3. Decomposition of a Multiply Connected Object

Decomposition of a multiply connected controlled object to n simply connected objects is provided by changing to a new basis of the state variables featured with decomposition matrix pair $(\hat{A}, \hat{B})$ where matrix $\hat{A}$ coincides matrix A, i.e.

$$\hat{A} = \begin{bmatrix} A_0 & \cdots & 0 \\ \cdots & \ddots & \cdots \\ 0 & \cdots & A_0 \end{bmatrix}_{(nm)\times(mn)} \,, \qquad (8)$$

and components $\hat{B}_i$ of the block column matrix $\hat{B} = [\hat{B}_1, \ldots, \hat{B}_n]^{\mathrm{T}}$ are the following:

$$\hat{B}_i = [0, \ldots, 0, \, \overline{b}_i \,, \, \ldots, \, 0]_{m \times n} \,, \qquad (9)$$

i.e. all the columns of matrix $\hat{B}_i$ must be zero excepting i-th column. In this case, it is reasonable to take vector $\overline{b}_i$ as $\overline{b}_i = [0, \ldots, 0, \, 1]^{\mathrm{T}}$.

Change of the basis of the state variables $\hat{\overline{x}}$ is provided by nondegenerate transformation

$$\hat{\overline{x}} = \Gamma \overline{x} \,, \qquad (10)$$

where $\Gamma = [\gamma_{ij}]_{mn \times nm}$ is a transfer matrix, $\det \Gamma \neq 0$.

So long as the original object is fully Kalman controllable, $A = \hat{A}$ and rank $B = n$, than, in accordance with [4], matrix Γ exists and is determined from equation

$$\Gamma = [\hat{B}, \hat{A}\hat{B}, \ldots, \hat{A}^{m-1}\hat{B}][B, AB, \ldots, A^{m-1}B]. \quad (11)$$

With provision for (6) – (11), equations of the multiply connected object in the new basis will be

$$\begin{aligned} \dot{\hat{\overline{x}}}_1 &= A_0 \hat{\overline{x}}_1 + \overline{b}_1 u_1 \,, \\ \cdots & \;\cdots \;\cdots \;\cdots \;\cdots \\ \dot{\hat{\overline{x}}}_n &= A_0 \hat{\overline{x}}_n + \overline{b}_n u_n \,. \end{aligned} \qquad (12)$$

Therefore multiply connected object of control fall apart into n subobjects having one input and not related to each other on the input actions u_i and on the state variables' vector $\hat{\overline{x}}_i \, (\dim \hat{\overline{x}}_i = m)$, $i = \overline{1,n}$. The vectors set forms the block vector $\hat{\bar{x}} = (\hat{\bar{x}}_1, \hat{\bar{x}}_2, \ldots, \hat{\bar{x}}_n)$ of the state variables of the multiply connected object in the new basis. It must be noticed the proposed decomposition approach provides "disentanglement" for the original multiply connected object on inputs and state variables only though its output (physical) coordinates are kept connected.

4. Relay Control Laws Synthesis

In the new basis of the state variables, the given task is decomposed to n more simple tasks of the relay control low synthesis for "disconnected" objects (12) under constraints (4). Herewith, in decomposition basis of the state variables, initial $\hat{\bar{x}}^{\mathrm{In}}$ and finite $\hat{\bar{x}}^{\mathrm{F}}$ conditions of the state variables vector are determined from equations

$$\hat{\bar{x}}^{\mathrm{In}} = \Gamma \bar{x}^{\mathrm{In}}, \quad \hat{\bar{x}}^{\mathrm{F}} = \Gamma \bar{x}^{\mathrm{F}}. \tag{13}$$

According to L.S. Pontryagin's principle of the maximum [5], the speed optimal actions $u_i(t)\,(i = \overline{1,n})$ are the following:

$$\begin{gathered} u_i(t) = a_i \operatorname{sign} k\big((\hat{x}_{i1}(t) - \hat{x}_{i1}^{\mathrm{F}}), \\ (\hat{x}_{i2}(t) - \hat{x}_{i2}^{\mathrm{F}}), \ldots, (\hat{x}_{im}(t) - \hat{x}_{im}^{\mathrm{F}})\big). \\ \hat{x}_{i1} = \hat{x}_{i1}^{\mathrm{In}}, \ \hat{x}_{i2} = \hat{x}_{i2}^{\mathrm{In}}, \ \ldots, \ \hat{x}_{im} = \hat{x}_{im}^{\mathrm{In}}, \end{gathered} \tag{14}$$

where $k\big((\hat{x}_{i1}(t) - \hat{x}_{i1}^{\mathrm{F}}), \ldots, (\hat{x}_{im}(t) - \hat{x}_{im}^{\mathrm{F}})\big)$ is a switch function; $\hat{x}_{i1}, \ldots, \hat{x}_{im}$ are components of the state variables vector $\hat{\bar{x}}_i$. The desired control low for the original basis of the state variables with provision for (10) and (13) will be

$$\begin{gathered} u_i(t) = a_i \operatorname{sign} k\left(\sum_{j=1}^{mn} \gamma_{m(i-1)+1,j}(x_j(t) - x_j^{\mathrm{F}}), \right. \\ \sum_{j=1}^{mn} \gamma_{m(i-1)+2,j}(x_j(t) - x_j^{\mathrm{F}}), \ldots \\ \left. \ldots, \sum_{j=1}^{mn} \gamma_{m(i-1)+1,j}(x_j(t) - x_j^{\mathrm{F}}) \right), \\ i = \overline{1,n}, \\ x_j(t_{\mathrm{In}}) = x_j^{\mathrm{In}}, \ j = \overline{1,mn}, \end{gathered} \tag{15}$$

where $x_j\ (j = \overline{1,mn})$ are components of vector $\bar{x}$.

Let's mention that in case of the various real characteristic numbers of matrix A_0 every of optimal control action (15) will have not more than $m-1$ switches (or m intervals of constancy). It follows from the feature of general solution of the differential equation system (3) and lemma about real zeroes of function [5].

Statement: If the controlled object follows equations (3) where matrix A is of the form of (5), characteristic numbers of matrix A_0 are various real numbers and control domain U is n-dimensioned cube (i.e. $|u_i| \le 1$ for every $i = \overline{1,n}$, then every of optimal actions $u_i(t)$ is piecewise-constant and has not more than $m-1$ switches (i.e. m intervals of constancy), where m is matrix A_0 dimension.

Proving: Let's consider vector of additional variables $\bar{\psi} = (\psi_1, \ldots, \psi_{mn})^{\mathrm{T}}$ satisfying the system of equations

$$\dot{\bar{\psi}} = -A^{\mathrm{T}} \bar{\psi} \tag{16}$$

which is adjoint to the system (3). According to L.S. Pontryagin's principle of the maximum, every component values u_i of control vector $\bar{u}$ providing maximum of Hamiltonian function (for the synthesis problem of the speed optimal control laws) are equal to [4]

$$u_i = \operatorname{sign} \sum_{j=1}^{mn} b_{ji} \psi_j, \quad i = \overline{1,n}, \tag{17}$$

where b_{ji} are coefficients of matrix B. Let's find general solution of adjoint system (16). As far as matrix A is block-diagonal of the form of (5), the system of equations (16) is the set of independent systems of homogeneous differential equations

$$\begin{gathered} \dot{\bar{\psi}}_1 = -A_0^{\mathrm{T}} \bar{\psi}_1, \\ \ldots \ \ldots \ \ldots \\ \dot{\bar{\psi}}_n = -A_0^{\mathrm{T}} \bar{\psi}_n, \end{gathered} \tag{18}$$

where vectors of additional variables $\bar{\psi}_1, \ldots, \bar{\psi}_n$ have dimension m and $\bar{\psi} = (\psi_1, \ldots, \psi_n)^{\mathrm{T}}$. Since characteristic numbers of matrix A_0 are various and real, the general solution of system (18) (and, therefore, of system (16)) will be of the form of

$$\psi_j(t) = \sum_{k=1}^{m} C_{jk} e^{-\lambda_k} k^t, \quad j = \overline{1,mn}, \tag{19}$$

where C_{jk} are some constant coefficients; $\lambda_1, \ldots, \lambda_m$ are characteristic numbers of matrix A_0. Let's find umber of real zeroes in the right-hand side of equation (17). Let's substitute (19) for ψ_j in equation (22). The result will be

$$\begin{aligned} u_i &= \operatorname{sign} \sum_{j=1}^{mn} b_{ji} \sum_{k=1}^{m} C_{jk} e^{-\lambda_k} k^t = \\ &= \operatorname{sign} \sum_{k=1}^{m} C_{jk}^{*} e^{-\lambda_k} k^t, \end{aligned} \tag{20}$$

where $C_{jk}^{*} = \sum_{j=1}^{mn} b_{ji} C_{jk}\ (i = \overline{1,n},\ k = \overline{1,m})$ are constant coefficients. According to lemma about real zeroes of function [5], the sum in the right-hand side of equation (20) has not more than $m-1$ switches (or m intervals of constancy) which was to be proved.

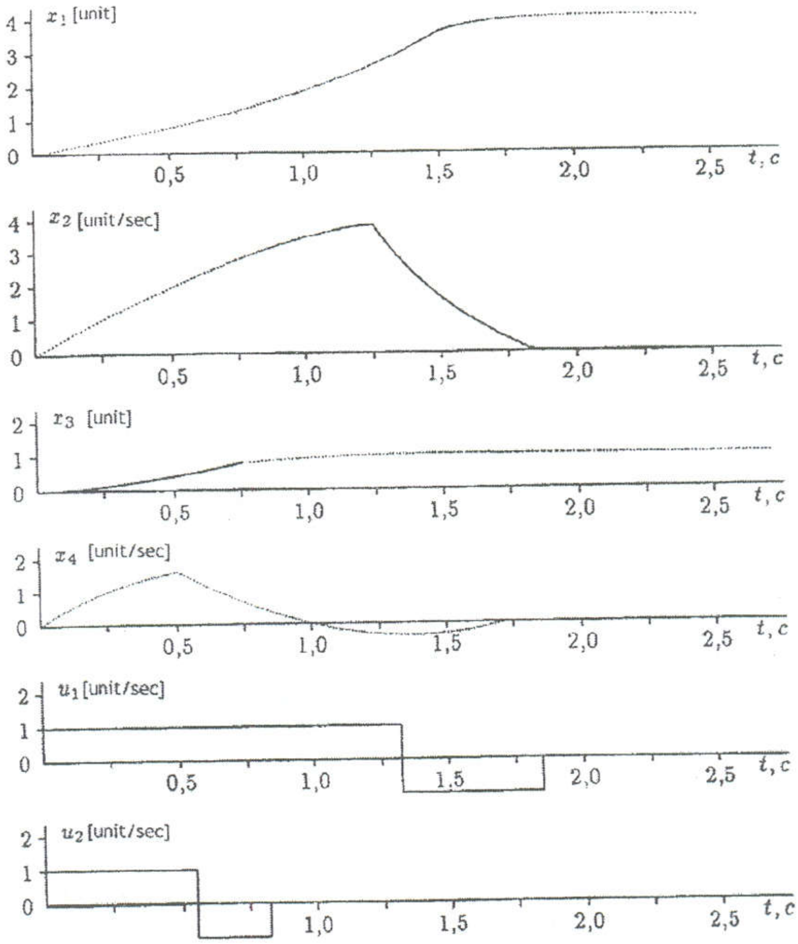

Figure 1. Changing of Control signals and State Variables

5. Example of the Control Laws Synthesis

Let's consider synthesis of the control laws for the group that consists of moving objects (MO). Relative motion of the objects is described with equation of the form of (3), herein matrixes A, B are the following:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}_{4\times 4}, \quad B=\begin{bmatrix} 0{,}1 & 0 \\ 5{,}0 & 0{,}1 \\ 0{,}1 & 0{,}2 \\ 0{,}2 & 4{,}0 \end{bmatrix}_{4\times 2}. \qquad (21)$$

It is seen from the formula (21) the dimension of polynomial $d(s)$ is equal to 2.

Let constraints on the control actions be assigned as

$$|u_1(t)| \le 1 \text{ and } |u_2(t)| \le 1. \qquad (22)$$

The components of the state coordinates vector $\overline{x}$ are relative coordinates of MO and speeds of its changing.

It is required to define the relay control law that takes vector of the state variables from the initial state $\overline{x}^{\text{In}}=(0,0,0,0)^{\text{T}}$ to the finite state $\overline{x}^{\text{F}}=(4,0,1,0)^{\text{T}}$ under constraints (22).

Let's decompose the original multiply connected object to two not related to each other subobjects with one input. In order to solve it let's choose decomposition matrix pair $(\overline{A},\overline{B})$ as the following:

$$\overline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}_{4\times 4}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}_{4\times 2}. \tag{23}$$

Than, in accordance with (11), let's calculate matrix Γ to change to the new basis of state variables

$$\Gamma = \begin{bmatrix} 0{,}1964 & -0{,}0038 & -0{,}0047 & 0{,}0003 \\ 0 & 0{,}2002 & 0 & -0{,}0050 \\ -0{,}0140 & -0{,}0040 & 0{,}2385 & -0{,}0118 \\ 0 & -0{,}0100 & 0 & 0{,}2503 \end{bmatrix}_{4\times 4}. \tag{24}$$

Control laws in original basis of the state variables in accordance with [4] and [5] are the following:

$$\begin{aligned} u_1 = {} & \operatorname{sign}[-0{,}1964((x_1-4)+x_2)+ \\ & +0{,}0047((x_3-1)+x_4)+ \\ & \ln(1+(0{,}2002x_2-0{,}0050x_4))\times \\ & \times\operatorname{sign}(0{,}2002x_2-0{,}0050x_4)], \\ u_2 = {} & \operatorname{sign}[0{,}0140((x_1-4)+x_2)+ \\ & +0{,}2385((x_3-1)+x_4)+ \\ & \ln(1+(-0{,}0010x_2-0{,}2503x_4))\times \\ & \times\operatorname{sign}(-0{,}0100x_2-0{,}2503x_4)]. \end{aligned} \tag{25}$$

Fig. 1 shows changing of the state variables $x_1 = z_1$, x_2, $x_3 = z_2$, x_4 and control signals u_1 and u_2.

From fig. 1, it is seen the constructed control laws provide changing of the relative coordinate of the first MO motion from 0 to 4 and of the second MO as from 0 to 1 in 1,75 seconds under satisfactory behaviour of z_1, z_2 during this time interval. Herein all the distances are measured in standard units (just "units" in what follows). Besides that, maximal variation of the motion speed for the first MO is 3,5 unit/sec, for the second MO – 2 unit/sec that is completely acceptable. Number of sign changing for the control actions u_1 and u_2 is the same and equal to 1 (the one less than dimension of polynomial $d(s)$ equal to 2).

6. Conclusion

Suggested approach to the control laws synthesis has the following features:

- Control for the group of autonomous moving objects aimed to provide a certain three-dimensional rank of the objects.
- Using relay laws belonging to speed optimal control laws.
- Multiply connected object decomposition to n simply connected objects with unrelated vectors of state variables to simplify the law synthesis.
- Solving n more simple tasks of the relay control low synthesis for "disconnected" subobjects.

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