Anode-Detail Shape Determination in Problems of Steady Electrochemical Machining at Use of the Polygonal Cathode-Tool

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Abstract[1]

The study implements a two-dimensional mathematical model of the ideal electrochemical machining (ECM) process at processing by the polygonal cathode-tool; the numerical - analytical method of the decision of a problem is submitted. In solving the problem of electrochemical forming, it is possible to use the method of hydrodynamic analogy, where the flat potential electrical field is replaced with a dummy flow of an ideal incompressible fluid. Then the considered problem is interpreted as a problem of the theory of jets in an ideal fluid. The allowance for the machining mode, electrolyte properties yields nonlinear conditions on borders at the free surface.

1. Introduction

Electrochemical machining (ECM) of metals is an advanced method for the production of workpieces from metals and alloys with a specified shape, size, and surface quality [1]. The method is based on the principle of the local dissolution of the anode – workpiece in the electrolyte flow. The cathode – machining tool – is represented by an electrode with a specified surface shape.

2. Model of the Process

For the first approximation in the theoretical analysis of the ECM the ideal process model is used. In an ideal process, the electric field can be described by the Laplace equation $\nabla^2 u = 0$, where u is the electric-field potential. The values of the potential u_a and u_c at the anode and cathode surfaces are constant [1]. The steady distribution of the current density i at the stationary anode boundary can be determined by the equality

$$\eta(i_a)\cdot i_a = \rho V_c \cos\theta/\varepsilon, \qquad (1)$$

Where η is the current efficiency coefficient, $i_a = \kappa\,\partial u/\partial n_a$ is the anode current density, κ is the specific electrical conductivity of the medium, ρ is the density of the anode material, ε is the electrochemical equivalent of the metal, θ is the angle between the vector of the cathode-feed velocity $\vec{V}_c$ and the unit vector of the outward normal to the anode $\vec{n}_a$. The dependence $\eta(i_a)$ is described by the hyperbolic equation [2]

$$\eta = a_0 + a_1/i_a\,. \qquad (2)$$

Here a_0, a_1 are constant coefficients.

Let us pass to the dimensionless variables $\psi = (u-u_c)/(u_a-u_c)$ and $n = n_a/H$. The characteristic length H is determined by the expression $H = \kappa(u_a-u_c)/i_0$. Where $i_0 = \rho V_c/\varepsilon$ is the characteristic current density.

The function ψ corresponding to the electric-field potential satisfies the Laplace equation $\nabla^2\psi = 0$ in the interelectrode gap. The following conditions are satisfied at the boundaries of the electrodes

$$\psi_a = 1, \quad \psi_k = 0\,. \qquad (3)$$

The steady condition is fulfilled at the anode boundary

$$\frac{\partial\psi}{\partial n} = \frac{1}{a_0}\left(\frac{a_1}{i_0} - \cos\theta\right) = a + b\cos\theta\,, \qquad (4)$$

where $a = a_1/(a_0\, i_0)$, $b = -1/a_0$.

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In formulation and solving problem, the hydrodynamic analogy of the electric field is employed, according to which plane potential electric field is replaced by a fictitious flow of an ideal incompressible fluid [3]. If we introduce the complex potential of the electrostatic field $W = \varphi + i\psi$, we obtain $\partial\psi/\partial n = V$ along the line $\psi = const$ (in the case of the hydrodynamic interpretation of the ECM problems, V is the velocity vector of the fictitious flow). In standard hydrodynamic terminology, the problem of anode-shape determination is called a free-boundary problem.

3. Problem Statement

3.1. The Problem Review and Basic Equations

In work the flat-parallel problem of steady electrochemical shaping is considered. The problem will consist in definition of the form of a detail which is formed at processing by the polygonal cathode-tool.

The possible scheme of an interelectrode gap is submitted on a figure 1. Here $ACDEB$ – border of the cathode, AB - anode boundary. Rectangular coordinates x and y are attached to the cathode. It is assumed that the cathode moves in the direction of the ordinate. Points A and B are in infinity.

Let's consider dummy flow of an ideal incompressible fluid. The region occupied the fluid is bounded by the free surface AB and the rigid boundary $ACDEB$.

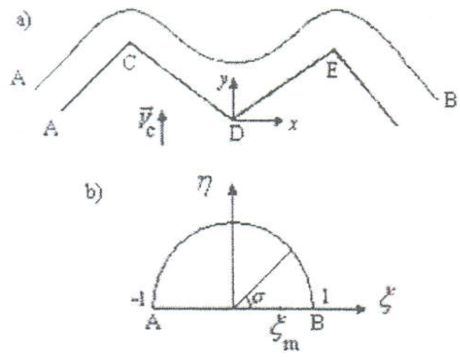

Figure 1. The Scheme the Interelectrode Gap in Different Planes: a) the Physical Plane; b) the Auxiliary Plane

To get solution we introduce the auxiliary complex variable $t = \xi + i\eta$, which varies in D_t $(|t| \le 1, \eta \ge 0)$, with the correspondence between the physical $z = x + iy$ plane and auxiliary semicircle $t = \xi + i\eta$ shown in figure 1, and to a free surface there corresponds an arch of a circle $t = \exp(i\sigma), \sigma \in (0, \pi)$. To points z_m on the rigid boundary in which the vector of velocity changes direction, there corresponded points ξ_m $(m = \overline{1, M})$.Then, we search for $z(t)$, which conformably maps area D_t onto the physical plane. For the solution of a problem it is necessary to find a derivative of the complex potential $W = \varphi + i\psi$ with respect to the auxiliary variable and Zhukovskii's function [4]

$$\chi(t) = \ln\frac{1}{V_0}\frac{dW}{dz} = \ln\frac{V}{V_0} - i\theta = r - i\theta, \quad r = \ln\frac{V}{V_0}, \quad (5)$$

Where: V is the module of the velocity, V_0 is the value of the module of the velocity in point B, θ is the angle of the inclination of the velocity vector to the x axis.

The values $\operatorname{Im} W(t)$ on the boundaries are the constants (4). Using the method of the singular points the derivative dW/dt can be defined [4].

The imaginary part of Zhukovskii's function on the rigid boundary accepts piecewise constant values:

$$\theta(\xi) = \theta_m, \quad \left(\xi_m < \xi < \xi_{m+1}, m = \overline{0, M}\right), \\ \xi_0 = -1, \xi_{M+1} = 1. \quad (6)$$

From formulas (4) and (5) follows

$$a + b\cos\theta(t) - V_0 \exp(r(t)) = 0, \\ t = \exp(i\sigma), \sigma \in [0, \pi]. \quad (7)$$

Where $V_0 = a + b\cos\theta(1)$, from here follows

$$r(1) = 0. \quad (8)$$

Function $\chi(u)$ is presented as the sum [5]

$$\chi(t) = \chi_*(t) + \omega(t) \quad (9)$$

Function $\chi_*(t)$ corresponds to auxiliary flow. Here the velocity absolute value on the free surface the constant and V_0 is equal. The function $\omega(t)$ is an analytic function in semicircle D_t and function $\omega(t)$ is continuous in $\overline{D}_t$. For function $\chi_*(t)$ the following boundary conditions satisfied

$$\operatorname{Im}\chi_*(\xi) = -\theta(\xi), \quad \xi \in [-1, 1], \\ \operatorname{Re}\chi_*(\exp(i\sigma)) = 0, \quad \sigma \in [0, \pi]. \quad (10)$$

The function $\chi_*(t)$ can be constructed by the method of the singular points [5]

$$\chi_*(t) = \ln\left(\prod_{m=1}^{M}\left(\frac{t - \xi_m}{t\xi_m - 1}\right)^{\alpha_m}\right) - i\theta(-1), \\ \alpha_m = (\theta_m - \theta_{m-1})/\pi, m = \overline{1, M}, \theta_0 = \theta(-1). \quad (11)$$

Where ξ_m are the points of discontinuity for function $\theta(\xi)$.

Applying designations $T = \operatorname{Im}\chi_{*}(\exp(i\sigma))$, $\mu = \operatorname{Im}\omega(\exp(i\sigma))$, $\lambda = \operatorname{Re}\omega(\exp(i\sigma))$, we shall write down boundary conditions for function $\omega(t)$

$$a + b\cos(T + \mu) - V_0\exp(\lambda) = 0, \qquad (12)$$

$$\operatorname{Im}\omega(\xi) = 0, \quad \xi \in [-1, 1], \qquad (13)$$

$$\operatorname{Re}\omega(1) = 0. \qquad (14)$$

Taking into account boundary conditions for function $\omega(t)$, we have [5]

$$\omega(t) = c_0 + \sum_{k=1}^{\infty} c_k t^k, \quad c_0 = -\sum_{k=1}^{\infty} c_k. \qquad (15)$$

The coefficient c_k is chosen so that the function $\chi(t)$ satisfies the boundary condition (12). The problem is reduced to solving equations (12). It is solved by the collocation method. After definition of coefficient c_k of series (15) all geometrical characteristics can be determined from the formula

$$z(t) = \frac{1}{V_0}\int \exp(-\chi(t))\frac{dW}{dt}dt + C. \qquad (16)$$

3.2. Anode-Detail Shape Determination in a Neighborhood of the Channel for Submission of Electrolyte

As an example we shall consider the following schema (fig. 2). The electrolyte moves in a working zone from the channel located in a body of the cathode.

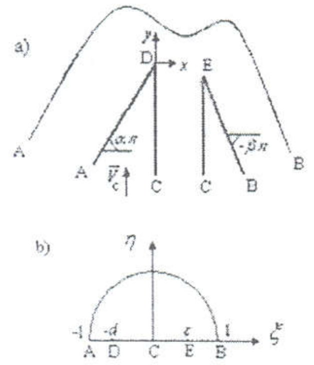

Figure 2. The Scheme the Interelectrode Gap in Different Planes: a) the Physical Plane; b) the Auxiliary Plane

Here $ADCEB$ - border of the cathode, AB - anode boundary. Lines CE and DE are borders of the channel. The width of the channel is equal L. The origin point of the Cartesian coordinate system is chosen in a point D. The ordinate of a point E is equal y_E. The vector $\vec{V}_c$ specifies the feed direction of the cathode. Points A, B and C are in infinity.

To get solution we introduce the auxiliary complex variable $t = \xi + i\eta$, which varies in D_t $(|t| \le 1, \eta \ge 0)$, with the correspondence between the physical $z = x + iy$ plane and auxiliary semicircle $t = \xi + i\eta$ shown in figure 2. Then, we search for $z(t)$, which conformably maps area D_t onto the physical plane. The function dW/dt can be constructed by the method of the singular points and looks like

$$\frac{dW}{dt} = \frac{4}{\pi(1 - t^2)}. \qquad (17)$$

According to (8), (10), we have

$$\chi(t) = \ln t - \left(\frac{1}{2} + \alpha\right)\ln\left(\frac{t + d}{1 + td}\right) - \left(\frac{1}{2} + \beta\right)\ln\left(\frac{t - \varepsilon}{1 - t\varepsilon}\right) + \beta\pi i + \omega(t). \qquad (18)$$

Where d, ε is the coordinates of points D, E in auxiliary semicircle, $\omega(t) = c_0 + \sum_{k=1}^{\infty} c_k t^k$, $c_0 = -\sum_{k=1}^{\infty} c_k$.

From formulas (17), (18) follows

$$z(t) = \frac{4\exp(\beta\pi i)}{\pi V_0}\int_0^t R(\tau, d, \varepsilon)\exp(-\omega(\tau))d\tau,$$

$$R(\tau, d, \varepsilon) = \frac{1}{\tau(1 - \tau^2)}\left(\frac{\tau - \varepsilon}{1 - \tau\varepsilon}\right)^{0.5+\beta}\left(\frac{\tau + d}{1 + \tau d}\right)^{0.5+\alpha}. \qquad (19)$$

For determination of mathematical parameters d, ε we have two equations:

$$L = \frac{4}{\pi V_0}\operatorname{Re}(F(d, \varepsilon)), \qquad (20)$$

$$y_E = \frac{4}{\pi V_0}\operatorname{Im}(F(d, \varepsilon)), \qquad (21)$$

where $F(d, \varepsilon) = \exp(\beta\pi i)\int_{-d}^{\varepsilon} R(\tau, d, \varepsilon)\exp(-\omega(\tau))d\tau$.

The unknown coefficient c_k are defined according to boundary condition (12).

4. Numerical Solution of the Problem

4.1. Numerical Algorithm

The collocation method [6] is used for numerical solution. A limited number of terms remain in the sum (15). The boundary condition (12) is fulfilled at the discrete points $\sigma_m = \pi m / n$, $m = \overline{1, n}$ of the circle arc $t = \exp(i\sigma)$. The system of the nonlinear equations for calculation of coefficient c_k is solved together with the equations (20), (21) for definition of mathematical parameters d, ε.

4.2. Computation Results

The calculations were performed for the following conditions. The characteristic current density was $i_0 = 100\, A/cm^2$, and the coefficients $a_0 = 0.906$ and $a_1 = -12.817$ corresponded to 5KhNM steel in 15% solution of $NaNO_3$. Geometrical parameters α, β, H, y_E vary.

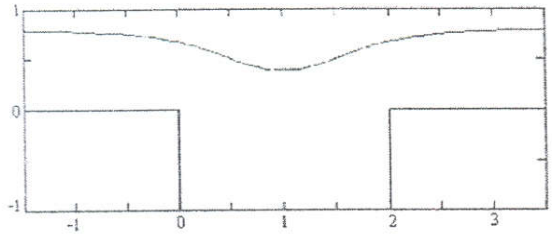

Figure 3. Calculated Anode Boundary at $\alpha = \beta = 0$, $L = 2$, $y_E = 0$

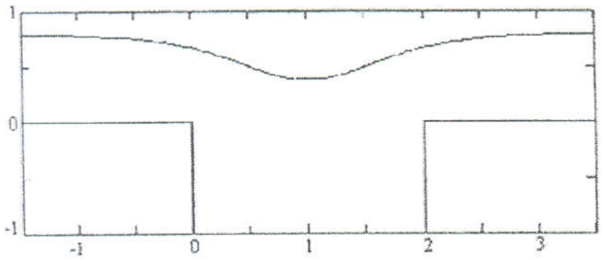

Figure 4. Calculated Anode Boundary at $\alpha = 0$, $\beta = 1/4$, $L = 2$, $y_E = 0$.

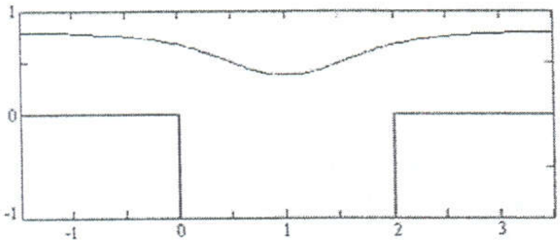

Figure 5. Calculated Anode Boundary at $\alpha = \beta = 1/3$, $L = 2$, $y_E = 0$.

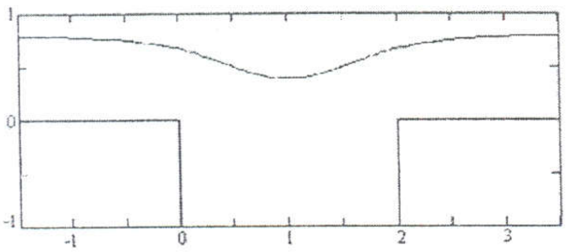

Figure 6. Calculated Anode Boundary at $\alpha = \beta = -1/3$, $L = 2$, $y_E = 0$.

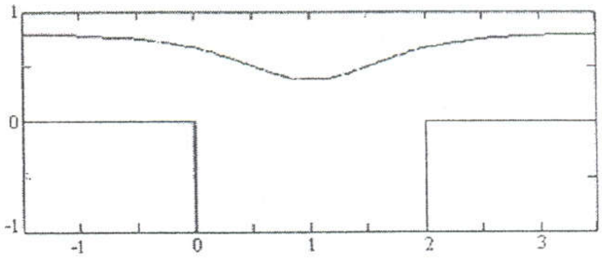

Figure 7. Calculated Anode Boundary at $\alpha = 1/3$, $\beta = 1/4$, $L = 2$, $y_E = 0$.

5. Conclusion

In work on the basis of two-dimensional mathematical model of the ideal process, the problem of definition of the form of study anodic boundary is solved at processing by the polygonal cathode-tool. The numerical simulation of a problem is executed for various special cases.

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