Modelling the Interrelation Macroeconomic Factors on the Market of Labour

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Abstract[1]

The focus of this paper is on results of computing experiment on construction and calculation of econometric models of the simultaneous structural equations establishing interrelations of macroeconomic parameters on the market of labour in Russia.

1. Introduction

Phillips's 1958 article had a considerable impact on macroeconomists and among macro policymakers. One very important paper by Robert E. Lucas, Jr. and Leonard A. Rapping [1970] reversed the causation and emphasized market-clearing rather than excess demand factors; in particular, Lucas and Rapping related an endogenous unemployment rate to expected wage inflation. A clear implication of their "backward" Phillips curve was that any observed relationship between inflation and unemployment must reflect the price expectations of firms and households; one should therefore not expect a Phillips curve to be stable unless these expectations are also stable.

An important contribution of the Lucas-Rapping paper was that it highlighted the role of expectations in interpreting the Phillips curve.

2. Publishing

Lucas and Rapping begin by assuming that when individuals make their consumption and leisure decisions, they do so in an intertemporal context. Specifically, assume that there are four arguments in the utility function of a representative household: current goods consumption C, current labor supply N, future goods consumption C^*, and future labor supply N^*. The household maximizes the utility function

$$U(C, C^*, N, N^*),$$
$$U_1, U_2 > 0,$$
$$U_3, U_4 < 0, \quad (1)$$

where the U_i indicate partial derivatives with respect to the sequence of arguments in $U(C, C^*, N, N^*)$. This utility function is maximized subject to the constraint that the present value of consumption cannot exceed the present value of income. The initial nonhuman assets are valued in money terms at A, the f nominal interest rate is r, and present and future goods prices and money wage rates are P, P^*, W, and W^*, respectively. Hence the budget constraint is written as:

$$PC + \left[(P^*/(1+r))C^*\right] \le A + WN + \left[(W^*/(1+r))N^*\right]. \quad (2)$$

Assuming the existence of a unique solution to this constrained maximization problem with positive prices, Lucas and Rapping write in implicit form the 1 current labor supply function, homogeneous of degree zero in its four arguments:

$$N = f\left[\frac{W}{P}, \frac{W^*}{P(1+r)}, \frac{P^*}{P(1+r)}, \frac{A}{P}\right]. \quad (3)$$

This simple theory of a single household suggests an aggregate empirical labor supply function relating total hours supplied annually N_t, divided by an index of the number of households M_t, to current and future money wages W_t and W^*_t, current and future prices (GNP deflators) P_t, and P^*_t a nominal interest rate r_t and the market value of assets held by the household sector A_t. Lucas and Rapping postulated an aggregate log-linear relationship

$$\ln(N_t/M_t) = \beta_0 + \beta_1 \ln(W_t/P_t) + \beta_2 \ln\left[W^*_t/(P_t(1+r))\right] +$$
$$+ \beta_3 \ln\left[P^*_t/(P_t(1+r_t))\right] + \beta_4 \ln\left[A_t/(P_t M_t)\right] \quad (4)$$

Proceedings of the 8th International Workshop on Computer Science and Information Technologies CSIT'2006
Karlsruhe, Germany, 2006

Denning real values with lowercase letters

$$w_t \equiv W_t / P_t,$$
$$w_t^* \equiv W_t^* / P_t^*,$$
$$a_t \equiv A_t / P_t,$$

letting $\beta_3 \equiv -(\beta_2 + \beta_3') > 0$ and noting that $\ln(1 + r_t) \approx r_t$ they rewrote Eq. (4) in the more familiar form

$$\ln(N_t / M_t) = \beta_0 + \beta_1 \ln w_t + \beta_2 \ln w_t^* + \\ + \beta_3 \left[r_t - \ln(P_t^* / P_t) \right] + \beta_4 \ln(a_t / M_t). \quad (5)$$

Equation (4) was modified for empirical purposes in several ways. First, since Lucas and Rapping expected the effects of real asset holdings on labor supply to be small, and since it was difficult to obtain reliable measures of a_t/M_t, they set $a_4 <= 0$ and excluded this regressor; also, although they reported some results with r_t included, Lucas and Rapping noted that "our most satisfactory models exclude this variable, and it will be dropped from the discussion that follows."

This left Lucas and Rapping with the problem of modeling the mechanism by which expectations of the real wage and prices were formed. They postulated the *adaptive expectations* scheme,

$$\frac{w_t^*}{w_{t-1}^*} = \left[\frac{w_t}{w_{t-1}^*} \right]^{\lambda} e^{\lambda'} \quad (6)$$

where λ is the adaptive expectations parameter, $0 < \lambda < 1$, and where $e^{\lambda'}$ is added to permit an anticipated trend in real wages. In this adaptive specification the greater the reliance of current expectations on the more recent past, the larger is λ; small values of λ imply a long memory, larger values a shorter recall. In logarithmic form, Eq. (6) becomes

$$\ln w_t^* - \ln w_{t-1}^* = \lambda(\ln w_t - \ln w_{t-1}^*) + \lambda, \quad (7)$$

or

$$\ln w_t^* = \lambda \ln w_t + (1 - \lambda) \ln w_{t-1}^* + \lambda'. \quad (8)$$

The adaptive expectations scheme can be viewed as an error correction process. Alternatively, if one employs repeated substitution procedures in Eq. (8), it becomes clear that with the adaptive expectations specification the unobservable expected wage rates are in fact a function of an infinite set of observable past rates of wage inflation.

Lucas and Rapping went on to assume that price anticipations are also formed adaptively, and that they had the same error correction parameter λ

$$\ln P_t^* = \lambda \ln P_t + (1 - \lambda) \ln P_{t-1}^* + \lambda'', \quad (9)$$

where the trend parameter λ was envisaged as depending on major political and military events as well as the past development of prices.

Deleting r_t and a_t/M_t as noted above, inserting repeated substitution versions of Eqs. (8) and (9) into Eq. (5), and then doing a Koyck transformation, Lucas and Rapping obtained the labor supply equation

$$\ln(N_t / M_t) = \left[\beta_0 \lambda + \beta_2 \lambda' - \beta_3 \lambda'' \right] + \\ + (\beta_1 + \lambda \beta_2) \ln w_t - (1 - \lambda) \beta_1 \ln w_{t-1} + \\ + (1 - \lambda) \beta_3 \ln(P_t / P_{t-1}) + \\ + (1 - \lambda) \ln(N_{t-1} / M_{t-1}), \quad (10)$$

which they then reparameterized as

$$\ln(N_t / M_t) = \beta_{10} + \beta_{11} \ln w_t - \beta_{12} \ln w_{t-1} + \\ + \beta_{13} \ln(P_t / P_{t-1}) + \beta_{14} \ln(N_{t-1} / M_{t-1}). \quad (11)$$

The economic theory underlying the derivation of this equation implied I four inequality restrictions on the parameters in Eq. (11), namely,

$$0 < \beta_{11} < \beta_{12} / \beta_{14}, \\ \beta_{12} > 0, \beta_{13} > 0, \\ 0 < \beta_{14} < 1. \quad (12)$$

For their labor demand equation, Lucas and Rapping employed a constant elasticity of substitution (CES) production function; owing to space constraints, we will not discuss this production relation in detail.Suffice it to say that Lucas and Rapping assumed that the CES production function was characterized by constant returns to scale, that firms chose inputs so as to maximize profits, and that input and output markets were competitive. Rearranging a logarithmic version of the marginal product of labor equals wage rate equation derived from the CES production function and permitting partial adjustment of firms labor demands to their long-run equilibrium Lucas and Rapping derived a labor demand equation having the form

$$\ln(Q_t N_t / Y_t) = \beta_{20} + \beta_{21} \ln(w_t / Q_t) + \\ + \beta_{22} \ln(Q_{t-1} N_{t-1} / Y_{t-1}) + \beta_{23} \ln(Y_t / Y_{t-1}), \quad (13)$$

where N_t and w_t are the quantities and real wage rate for labor, Y_t is the level of output in constant dollars, and Q_t is an index of labor quality, which in practice is a years-of-schooling-completed index. In terms of parameters, β_{21} represents the negative of the short-run elasticity of substitution between capital and labor inputs, and β_{22} and β_{23} reflect the effects of the gradual adjust ment of employment and output to long-run equilibrium.

The third and final equation of the Lucas-Rapping model is of special importance because it relates changes in the measured unemployment rate to inflation. Lucas and Rapping explicitly assumed that the current wage rate equates labor supply with labor demand. But from where does unemployment emerge in such an equilibrium

model? To allow for the existence of measured unemployment consistent with their equilibrium view of the labor market, Lucas and Rapping offered an alternative hypothesis about what people mean when they classify themselves as unemployed.

Lucas and Rapping finally obtained their inverted Phillips curve, in which measured unemployment is a function of price and wage inflation:

$$u_t = \left[\gamma_0\lambda + \gamma_1\beta_1\lambda' + \gamma_1\beta_3\lambda''\right] - \gamma_1\beta_1 \ln(w_t / w_{t-1}) - \\ - \gamma_1\beta_3 \ln(P_t / P_{t-1}) + (1-\lambda)u_{t-1}, \quad (14)$$

which they reparameterized as

$$u_t = \beta_{30} - \beta_{31}\ln(w_t / w_{t-1}) - \beta_{32}\ln(P_t / P_{t-1}) + \beta_{33}u_{t-1} \quad (15)$$

Several features of Eq. (15) merit comment. There is an expected short-run negative relationship between inflation and unemployment; it is noteworthy that Lucas and Rapping were ablet to derive this Phillips curve relation from a framework emphasizing labor market equilibrium. Equation (15) does not necessarily imply a long-term- tradeoff, however; that depends on how the wage and price expectations terms adjust to experience.

Second, Lucas and Rapping argued that since the trend rates of real wages λ' and prices λ" appear in the constant term of Eq. (14), one should not expect the Phillips curve to be stable whenever economies experience sharp changes in wages and prices; given sufficient cause, firms and households will eventually revise their expectations accordingly.

The reporting results based on OLS estimation of the reduced form equations and from 2SLS estimation of the labor supply equation (11), the labor demand equation (13), and the measured unemployment equation (15), all using aggregate annual Russian time series data from1992 to 2005. The results for the three structural equations is as follows.

Labor supply:

$$\ln(N_t / M_t) = \underset{(0,24)}{-0,61} + \underset{(0,04)}{0.15}\ln \hat{w}_t - \underset{(0,05)}{0,01}\ln w_{t-1} + \\ + \underset{(0,02)}{0.03}\ln(P_t / P_{t-1}) + \underset{(0,31)}{0.48}\ln(N_{t-1} / M_{t-1}); \quad (16)$$

$$\bar{R}^2 = 0.870; DW = 2,26.$$

Labor demand:

$$\ln(N_t Q_t / Y_t) = \underset{(0.76)}{2,50} - \underset{(0.05)}{0.11}\ln(\hat{w}_t / Q_t) + \\ + \underset{(0.16)}{0.46}\ln(N_{t-1}Q_{t-1} / Y_{t-1}) - \underset{(0.08)}{0.57}\ln(Y_t / Y_{t-1}); \quad (17)$$

$$\bar{R}^2 = 0.942; DW = 1.54.$$

Unemployment:

$$u_t = \underset{(1,24)}{2,33} - \underset{(0.81)}{0.17}\ln(P_t / P_{t-1}) - \underset{(1,36)}{5,70}\ln(\hat{w}_t / w_{t-1}) + \underset{(0.12)}{0.77}u_{t-1};$$

$$\bar{R}^2 = 0.874; DW = 2.28 \quad (18)$$

3. Conclusion

The received model is close enough to the model calculated by Lucas and Rapping on the data of USA 1930 -1965. Concurrence of signs on factors of regress indicates similar laws of functioning on the market of labour. One of the important conclusions from the given model consists that attempts of the government move along Phillips's curve may be destroyed by expectations of people and to lead to changes which will shift a curve and will make any policy inefficient.

References

1. Berndt E.R. "The practice of econometrics: classic and contemporary", 2002, pp. 599-698.