LP-based exact approach for the 1D contiguous cutting-stock problem

M.A. Mesyagutov
Mathematics department
Ufa state aviation technical university
Ufa, Russia
e-mail: mmesyagutov@googlemail.com

Abstract[1]

In this paper the 1D contiguous cutting-stock problem is considered. An exact approach based on LP bounds for the mentioned problem is proposed. Also, there are numerical experiments.

1. Introduction

Given a set of indexes $I = \{1,\dots,m\}$ of rectangular items, each having width w_i and height $h_i (i \in I)$, and a strip of width W and infinite height, the two-dimensional strip-packing problem (2SP) consists of orthogonally packing all the items, without overlapping, into the strip by minimizing the overall height of the packing. We assume that the items have fixed orientation, i.e., they cannot be rotated.

Let us consider a problem which is closely related to the latter. Given an instance of 2SP we formulate the following relaxed problem. Consider, for each $i \in I$, h_i items of size $w_i \times 1$. For each $w_i \times 1$, all items have to be packed contiguously in the H-direction in order to find a packing which minimizes the height. Each item can be packed only once into one stock. This problem is known as the one-dimensional contiguous stock-cutting problem (1CSC).

Thus, each feasible solution of 2SP is a feasible solution of the corresponding 1CSC problem, but not vice versa. An optimum of 2SP is not necessarily an optimum of the corresponding 1CSC. According to the definition of 1CSC, the height of an optimum (or any lower bound for the latter) of 1CSC is also a lower bound of the corresponding 2SP. The domination of this bound over material bound is obvious. The 1CSC problem has also many applications, e.g., in scheduling.

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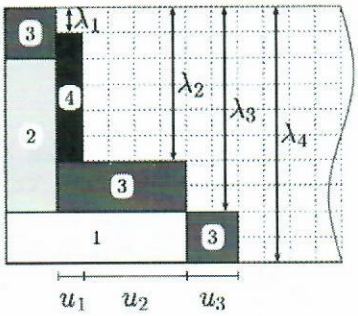
2. Exact approach

We propose an exact approach to solve 1CSC. The approach is based on a branch & bound method on item sequence permutations with different search strategies. It is quite similar to the one from [2] where 1CSC (called 1CBP, 1D contiguous bin packing) was used as a lower bound for 2SP. The branching is organized in such a way that each node is divided into subproblems according to the following principle. Each child differs from the parent that in the child node one of the item types $i \in I$ is fixed in some free position. Thereby, the number of children of each node is equal to the number of free items, i.e., the root node contains m children, nodes of depth one contain $m-1$ children, etc.

For the calculation of a lower bound we use the continuous relaxation of the one-dimensional cutting-stock problem with multiple stock lengths (1CSPM).



Let a partial solution be given where rectangles $\widetilde{I}$ are already packed. The unused area with the width of λ_k, and the height of u_k corresponds to a set of size u_k of stock materials with the width λ_k and unit height. Thus, at the disposal of the remaining rectangles $\overline{I} = I \setminus \widetilde{I}$ for cutting, we have q types of stock materials with widths λ_k where the largest stock λ_q has width equal to W, and is available in unlimited quantity. Thereby, a cutting pattern of one single stock length can be represented as

$$a^{jk} = (a_{1jk}, \dots, a_{|\overline{I}|jk})^T \in \{0,1\}^{|\overline{I}|}$$

with $\sum_{i \in \overline{I}} w_i a_{ijk} \le \lambda_k$. The relaxed model of 1CSPM is:

$$z^{qCSP} = \sum_j x_{jq} \downarrow \min, \text{ subject to}$$

$$\sum_k \sum_j a_{ijk} x_{jk} \ge h_i, \quad i \in \overline{I}$$

$$\sum_j x_{jk} \le u_k, \quad k = \overline{1, q-1}$$

$$x_{jk} \in \Re_+, \quad \forall j, k$$

For the solution of the latter the column generation method with binary knapsack problem is used, cf. [4]. The number $\widetilde{h} + \sum_{k=1}^{q-1} u_k + \lceil z^{qCSP} \rceil$ is used as a lower bound for the residual problem $\overline{I}$.

The approach is based on permutations of the item sequence and can produce many symmetrical solutions. In the first place there are many vertical and horizontal symmetries. To eliminate them we propose dominance criteria. We propose also a preprocessing procedure which is executed before branching. The procedure lets us find a subset of items which can be cut in only one optimal way.

Several strategies of search are considered. We combine best bound search and a diving procedure. In problems in which the optimum can not be found in acceptable time we use best bound search in order to improve the lower bound. This approach is different to [1] where depth-first search was used.

3. Numerical experiments

Numerical experiment was made on 2D instances of Bortfeld library [3, 5]. 209 instances were solved optimally. Since Bortfeld has given a lower bound for each problem, we can compare it with an exact solution of the contiguous problem. In the table the average absolute values of lower bounds are shown.

Класс	m	Bortfeld	1CSC
cl01	50	187,18	187,14
cl02	50	60,52	60,52
cl03	50	504,12	507,84
cl04	50	193,50	193,50
cl05	50	1613,16	1631,72
cl06	50	506,40	506,40
cl07	50	1577,82	1591,24
cl08	50	1397,92	1399,84
cl09	50	3343,10	3346,16
cl10	50	909,16	917,70
	500	1029,29	1034,27

4. Conclusion

In this paper was proposed a new lower bound for 2SP with branchings. This lower bound is accepted to be used in 2D exact methods to prune the enumeration. It can be also used for estimation of the quality of heuristics.

References

1. Martello S., Monachi M., Vigo D. "An exact approach to the strip-packing problem". *INFORMS Journal on Computing*, 2003; 15(3): 310-319.
2. Monachi M. "Algorithms for packing and scheduling problems". *PhD Thesis*. Università di Bologna, 2001.
3. Lodi A., Martello S., Vigo D. "Recent advance on two-dimensional bin packing problems". *Discrete Applied Mathematics*, 2002; 123: 379-396.
4. Caprara A., Monaci M. "Bidimensional packing by bilinear programming". *Mathematical Programming*. Ser. A. Published online, 2007.
5. Bortfeld A. "A generic algorithm for the two-dimensional strip packing problem with rectangular prices". *European Journal of Operational Research*, 2006; 172: 814-837.