Mathematical Models of Hydrate Formation in Gas Wells

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Abstract[1]

In the frame work of tube hydraulics quasi-stationary model of natural gas hydrates formation (dissociation) in gas wells has been proposed. The corresponding algorithm of its realization is based on the forth order Runge - Kutta method and on the iteration procedure which gives way to unite the problem of calculation of pressure and temperature during gas flow in well with changing cross-section with the problem of hydrate layer dynamics. Some results of calculations are given where one can see the influence of model parameters and initial conditions on dynamics of hydrate plug formation. These calculations are based on real cases related to gas fields of Eastern Siberia.

The problem statement and solution

The quasi-stationary mathematical model of hydrate formation in gas wells and pipelines was proposed in [1]. In the model real gas flow was described in the frame work of tube hydraulic while dynamics of hydrate formation was formulated as generalized Stefan problem in which phase transition temperature was a function of gas pressure. It was shown that the corresponding system of differential equations could be reduced to the following set:

$$\frac{dp}{dx} = -\rho g \sin\varphi - \frac{\sqrt{\pi}\psi M^2}{4\rho S^{2.5} S_0^{2.5}}, \qquad (1)$$

$$\frac{dT}{dx} - \varepsilon\frac{dp}{dx} = \frac{\pi D\alpha}{c_p M}(T_e - T) - \frac{g}{c_p}\sin\varphi, \qquad (2)$$

where ρ, c_p – density and specific heat capacity of gas; g – gravity acceleration; S, D – tube cross-section and diameter, correspondingly; x – distance along tube axes; p – pressure; φ – tube inclination to horizontal surface; ψ – hydraulic resistance; T_e – environments temperature; T – gas temperature; α – total heat transfer coefficient; $M = \rho v S$ –mass flow rate, constant; v – gas velocity; zero indices mean initial state.

Gas density is connected with pressure and temperature according to the equation of state:

$$\rho = \frac{p}{zRT},\ z = 1 + 0.07\frac{p}{T}\left(1 - \frac{6}{T^2}\right),\ \varepsilon = \frac{RT^2}{c_p p}\frac{dz}{dT}, \qquad (3)$$

where R – gas constant, compressibility coefficient z is defined by Berthelot equation, and throttle coefficient ε is calculated according to the third formulae in (3). Note that in the second formula there pressure and temperature are divided by their critical values which depend on natural gas composition.

In recent publication [2] the model was generalized in the following way. Bottom-hole temperature and pressure instead of being given were calculated from the solution of non-isothermal problem of gas flow in gas reservoir [3], while temperature in rocks around a well was calculated from the numerical solution of heat conduction equation.

In publication [1] as well as in some newer ones, for example in [2, 3, 4], the equation which describes dynamics of free cross section in a well S, is formulated for constant value of coefficient of heat transfer between gas and the inner surface of hydrate layer:

$$\frac{dS}{d\tau} = b_2\frac{T_e - T_{ph}(p)}{1 - b_2 \ln S} - b_1\sqrt{S}(T_{ph}(p) - T) \qquad (4)$$

where $b_1 = \frac{\alpha_1 D_0}{4\lambda_h}$, $b_2 = \frac{\alpha_2 D_0}{4\lambda_h}$, α_1 – coefficient of heat transfer between gas and hydrate layer; α_2 – coefficient of heat transfer between hydrate layer and rocks; λ_h – heat conductivity of hydrate; dimensionless time $\tau = \frac{\lambda_h T_c}{\rho_h l_h D_0^2} t$, T_c – critical temperature of gas, ρ_h – hydrate density, l_h – specific latent heat of hydrate formation; $T_{ph}(p) = a\ln p + b$ – equilibrium dissociation temperature where empirical coefficients a, b depend on gas composition.

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Initial conditions for equations (1), (2) и (4):

$$p(0)=p_0,\quad T(0)=T_0,\quad S(0)=S_0 \qquad (5)$$

In the present paper the model modified to take into account that this coefficient α_1 depends on changing with time free cross section of a tube. To derive the necessary relation we use the well-known semi empiric formulae for heat transfer coefficient from turbulent fluid flow to inner wall of a tube [5]:

$$Nu = 0.023 Pr^{0.43} Re^{0.8} \qquad (6)$$

where $Nu=\dfrac{\alpha_1 D}{\lambda_g}, Pr, Re=\dfrac{vD\rho}{\mu}$ – numbers of Nusselt, Prandtl and Reynolds, correspondingly; μ and λ_g – dynamic viscosity and heat conductivity of gas.

Now bearing in mind a relation for mass flow rate we derive from (6):

$$\frac{\alpha_1 D_0}{\lambda_g} = 0.023 Pr^{0.43}\left(\frac{M_0}{D_0\mu}\right)^{0.8}\left(\frac{\pi}{4}\right)^{0.1}\frac{M^{0.8}}{S^{0.9}} \qquad (7)$$

It is also necessary to change algorithm of calculations: in those sections of a well where hydrate layer exists, heat transfer coefficient in equation (2) should be calculated by the formulae (7) and simultaneously rock temperature T_e should be replaced with T_{ph}, that is phase transition temperature.

Calculations have been performed at the two sets of parameters where the first one corresponds to the Middle Vilui field in Yakutia, while the second one to Messoyakha field in Krasnoyarsk region (all figures are in SI):

1) $\alpha=5.82$, $R=520$, $D=0.1$, $\psi=0.02$, $\rho_h=920$, $l_h=510000$, $\lambda_h=1.88$, $\lambda_g=0.0307$, $c_p=2300$, $\mu=1.3\cdot10^{-5}$, $Pr=0.886$ (dimensionless), $p_0=240\cdot10^5$, $T_0=323$, $T_c=205.239$, $p_c=46.573\cdot10^5$, $L=2550$, $T_{e0}=328$, $\Gamma=0.0256$, $T_e=\begin{cases}T_{e0}-\Gamma x, & 0<x<L-400\\ 271.15, & L-400<x<L\end{cases}$, $a=7.009$, $b=178.28$.

2) $p_0=66\cdot10^5$, $T_0=282.91$, $T_c=191.202$, $p_c=46.893\cdot10^5$, $L=900$, $T_{e0}=283.15$, $\Gamma=0.024$, $a=10.036$, $b=126.023$, the rest of data here are equal to those in the first set.

Before analyzing the results of calculations we should say that initially the optimal mass flow rate for each case has been determined. It corresponds to the minimum of heat losses in wells while they are free from hydrates. For the Middle Vilui field it appears to be 4.5 kg/s, for Messoyakha field – 2.5 kg/s. In the following numerical experiment we study the effect of mass flow rate and initial values of free cross section on its dynamics.

At first let us analyze the results for the first set of data. Fig.1 and Fig.4 show the dynamics of free cross section of a well, while Fig. 2 and Fig. 3 – changes of temperature along the well. When initially a well is free from hydrates, that is $S(0)=1$, the more is mass flow rate the less is duration of the period of full well blockage. For $M=4.5$ it is equal to approximately 13 days while for $M=2.0$ – 25 days. The result is in agreement with expression (7): according to it heat transfer coefficient increases with increasing of mass flow rate. However in both cases the depth of hydrate formation zone is approximately the same. It can be clearly seen if one compares corresponding curves on Fig.2 and Fig.3: for bigger mass flow rate the point of intersection of gas temperature (lines 2 and 3) with equilibrium dissociation temperature lies at the depth of 2550-1718=832 m, for lower one at the depth of – 2550-1668=882 m. Fig.2 and Fig.3 also show that as well is being blocked gas temperature essentially lowers which is due to pressure drop near well head (see lines 2 in Fig.2 and Fig.3). Physically it corresponds to sharp decrease of free cross section while mass flow rate remains constant.

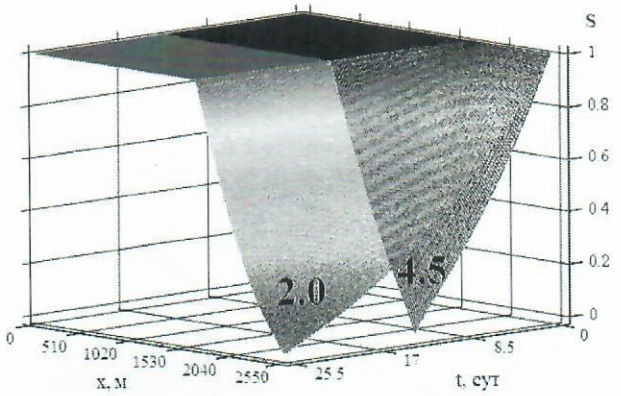

Fig.1. Free cross-section of well on the Middle-Vilui field vs. distance from the bottom and time at $S(0)=1$ (figures on maps correspond to mass flow rate).

Dynamics of free cross section becomes more complex if initially a well is partially blocked by hydrates. Fig.4 shows that in the lower part of well from bottom-hole to the depth of about 2550-1550=1000 m free cross section increases with time and finally this part of a well becomes clear from hydrates. But above this depth hydrate layer grows and blocks well in 2.7 days when mass flow rate is equal to 4.5 kg/s, and in approximately 8 days when mass flow rate is equal to 2.0 kg/s. These two regions are divided by the line of intersection of horizontal plane $S=0.5$ with the surface $S(t,x)$.

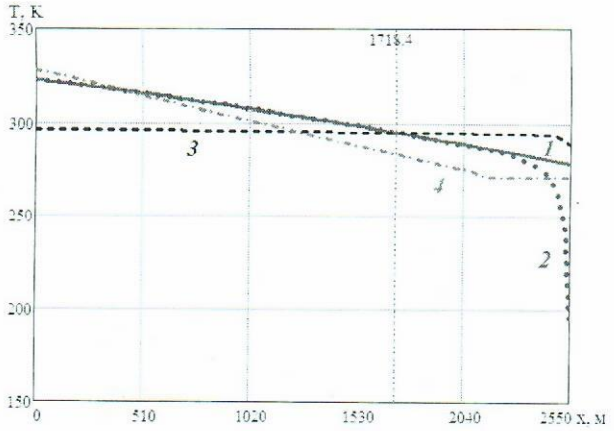

Fig. 2. Gas temperature vs. distance from the bottom for the Middle-Vilui field for M = 4.5 kg/c (1 – t = 3 hours; 2 – t = 13 days; 3 – equilibrium dissociation temperature; 4 – rock temperature)

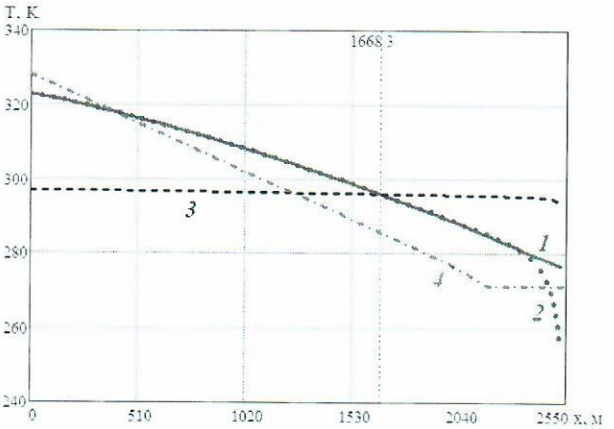

Fig. 3. The same as on Fig.2 for M = 2.0 kg/c (line 2 correspond to 25 days)

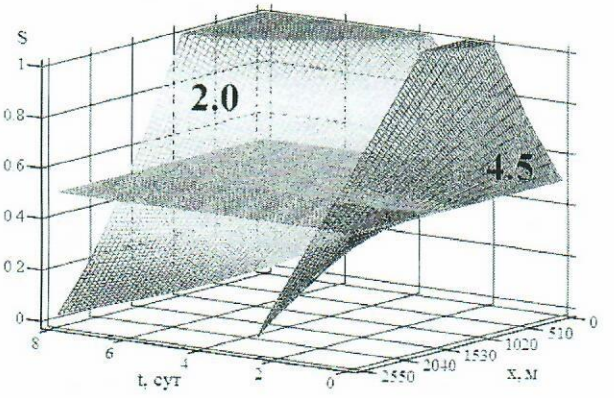

Fig.4. The same as on Fig.1 at $S(0)$ = 0.5

Now consider the results of calculations for the second set of data (Fig. 5–7). This set differs from the first one due to much shallower depth of reservoir (only 900 m), and correspondingly lower pressure and temperature, though the depth of is about the same. Calculations have been performed for two values of mass flow rate: 1.0 kg/s and 2.5 kg/s. Fig.5 and Fig.7 show dynamics of free cross section of a well. One can see that when initially hydrate layer is absent then will occur in 39 days at mass flow rate 2.5 kg/s and in 93 days at mass flow rate 1 kg/s (Fig.5). It means that for the second data set the complete well blockage requires much longer time interval than in the first example. Here the point of intersection of gas temperature and equilibrium dissociation lines correspond to the depth of 900-242=658 m, which is much lower than the depth of permafrost (Fig.6).

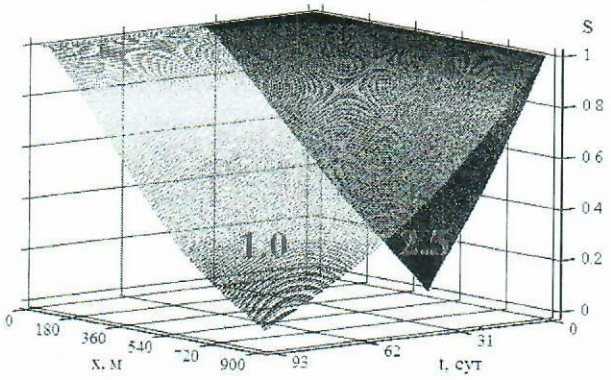

Fig. 5. Free cross-section of well on the Messoyakha field vs. distance from the bottom and time at $S(0)$ = 1 (figures on maps correspond to mass flow rate)

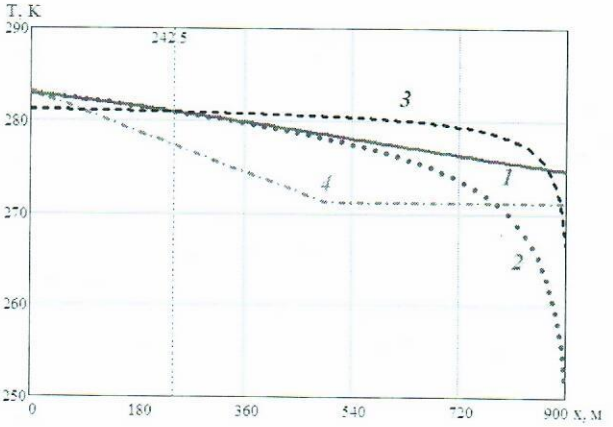

Fig.6. Gas temperature vs. distance from the bottom for the Messoyakha field for M = 2.5 kg/c (1 – t = 3 hours; 2 – t = 39 days; 3 - equilibrium dissociation temperature; 4 – rock temperature)

The results for these two sets differ even greatly when initially a well is partly blocked by hydrate (Fig. 7). The process of hydrate formation ceases in 6 days at mass flow rate 2.5 kg/s and in approximately 28 days at mass flow rate 1 kg/s. Moreover, for bigger mass flow rate a well is not blocked completely: about 15% of cross section remains free from hydrates. However the major difference from the first case is that here the well interval where hydrates dissociate, that is where free cross section increases with time, is practically absent (compare Fig. 4 and Fig. 7). The explanation lies in low bottom-hole temperature which is very close to equilibrium temperature of hydrate formation.

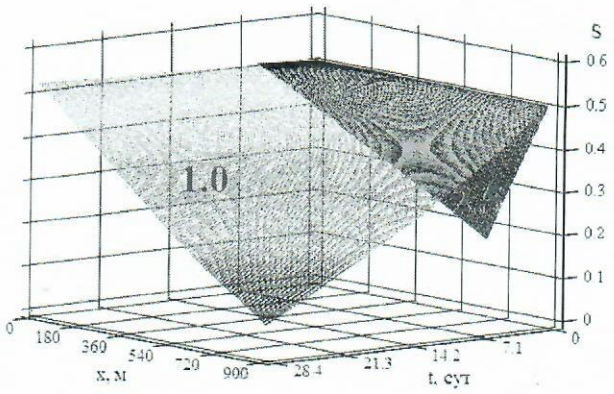

Fig.7. The same as on Fig.5 at $S(0) = 0.5$

Conclusion

The presented results reveal complexity of hydrate formation in gas wells. Its prediction and therefore, reliable gas production are possible only when one takes into account the whole combination of such factors as production rate, gas composition, well depth, bottom-hole temperature, geothermic conditions and a well characteristics before its start up. However they also reveal that period of hydrate formation in wells, even at low reservoir temperature and great depth of permafrost, is sufficiently long to prevent a possible emergency situation in a gas supply systems. Meanwhile it is clear that mathematical models of such a complex event should also be rather complex to take into account the most important physics of the process.

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